FRICTION AND HEAT TRANSFER WITH SIMULTANEOUS CONVECTION ON A PERMEABLE SURFACE

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The velocity and heat transfer fields near a vertical permeable surface with simultaneous convection are investigated. A solution is found for the boundary layer equations with known laws of surface temperature and flow velocity change. The transformed boundary layer equations contain the parameter G/R^2, which determines the effect of free convection on friction and heat transfer for constrained motion. Calculations of friction and heat transfer as functions of draft (suction) with simultaneous convection are presented.

The study of convective heat transfer in the literature is limited to an examination of constrained motion, where the rates of momentum transfer are large, or to natural convection, where there is a temperature drop between surface and medium. Little-studied flows with low velocity and temperature difference, ensuring the action of lifting forces and the free motion accompanying them, which produces an effect on heat transfer and tangent pressure, are also of interest.

The velocity and temperature fields near a vertical surface are described by the fundamental laws of conservation of mass, momentum, and energy. The differential equations of an incompressible laminar boundary layer describing convection for constant physical parameters of the medium with the exception of density, which is temperature dependent in a term expressing free motion, neglecting viscous dissipation, have the form

$$\begin{gathered}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{dp}{dx}+g\beta_T(T-T_\infty)\\ u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}\end{gathered}\tag{1}$$

with boundary conditions

$$\begin{gathered}u=0,\ v=v_w,\ T=T_w\ \text{for}\ y=0\\ u=U_\infty,\ T=T_\infty\ \text{for}\ y\to\infty\end{gathered}\tag{2}$$

Equation (1) is written for a coordinate system with x axis directed along the surface upward, and y axis normal thereto; in Eqs. (1) and (2) u and v are the axial components of the velocity, ν is the kinematic viscosity, p is pressure, T is temperature, a is the thermal diffusivity coefficient, g is the acceleration of gravity, β_T is the coefficient of thermal expansion, and the indices w and ∞ indicate values at the surface and the outer edge of the boundary layer.

For definiteness, we will assume that the surface temperature is greater than that of the medium ($T_w > T_\infty$). We will examine the motion when flow velocity and surface temperature are given by the expressions

$$U_\infty = Cx^m \tag{3}$$

$$T_w - T_\infty = Bx^n \tag{4}$$

where C, B, m, and n are constants.

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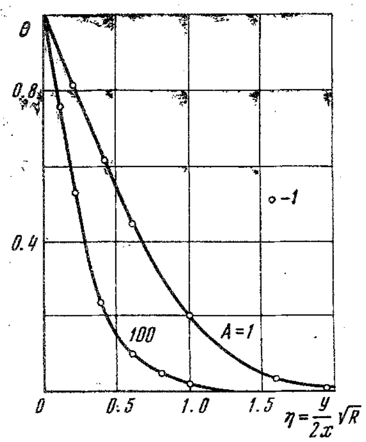

Fig. 1

With consideration of these expressions we solve Eq. (1), reducing the equations to ordinary differential equations. The reduction to ordinary differential equations with convection is performed with the aid of the independent variable

$$\eta = C_1 y x^{\beta} \tag{5}$$

and the flow function

$$\Psi = C_2 x^{\alpha} f(\eta) \tag{6}$$

such that the conditions

$$u = \frac{\partial \Psi}{\partial y}, \qquad v = -\frac{\partial \Psi}{\partial x} \tag{7}$$

are fulfilled.

In Eqs. (5) and (6) C_1, C_2, α, and β are unknown values, requiring determination.

Defining the velocity components according to Eq. (7)

$$\begin{gathered} u = C_1 C_2 x^{\alpha+\beta} f'(\eta) \\ v = -C_2 x^{\alpha-1}[\alpha f(\eta) + \eta\beta f'(\eta)] \end{gathered} \tag{8}$$

together with their derivatives, and substituting in Eq. (1), after simple transformations we obtain equations in dimensionless form

$$f'''(\eta) + (m+1) f(\eta) f''(\eta) - 2mf'^2(\eta) + 8\left[m + \frac{G}{R^2}\theta(\eta)\right] = 0 \tag{9}$$

$$\theta''(\eta) + P\left[(m+1) f(\eta)\theta'(\eta) - (4m-2) f'(\eta)\theta(\eta)\right] = 0 \tag{10}$$

where $\theta(\eta) = (T - T_\infty)/(T_W - T_\infty)$ is the dimensionless temperature,

$$G = \frac{g\beta_T (T_w - T_\infty) x^3}{\nu^2}, \quad R = \frac{U_\infty x}{\nu}, \quad P = \frac{\nu}{a}$$

are the Grashof, Reynolds, and Prandtl numbers, and the prime denotes differentiation with respect to η.

The boundary conditions in the new variables take on the form

$$\begin{gathered} f(0) = 0, \ f_w = \text{const}, \ \theta = 1 \ \text{ for } \ \eta = 0 \\ f'(\infty) = 2, \ \ \theta = 0 \ \text{ for } \ \eta \to \infty \end{gathered}$$

A transformation to ordinary differential equations is possible if

$$n = 2m - 1 \tag{11}$$

and the unknown values in Eqs. (5) and (6) are defined as follows:

$$\begin{gathered} \alpha = (m+1)/2 = (n+3)/4, \ \ \beta = (m-1)/2 = (n-1)/4 \\ C_1 = 0.5\,(C/\nu)^{0.5}, \quad C_2 = (C\nu)^{0.5} \end{gathered}$$

The boundary condition f_W = const, which is necessary for the transformation, indicates that $v_W \sim x^{(m-1)/2}$ [1, 2]. From the second equation of Eq. (8) we find the draft (suction) parameter

$$f_w = -\frac{2v_w}{(m+1)U_\infty}\sqrt{R}$$

In the transformed motion equation (9) the parameter $G/R^2 = A$ appeared, reflecting the effect of free convection on friction and heat transfer. For $A = 0$, Eqs. (9) and (10) describe constrained convection.

The system of ordinary equations (9) and (10) is solved by the numerical iteration method, eliminating the boundary problems which occur at each iteration [3]. As a result, dimensionless velocity and temperature profiles in the boundary layer with convection and draft (suction) are obtained for $P = 0.7$, $m = 0.5$, and $n = 0$.

In Fig. 1 dimensionless temperature profiles in the boundary layer with convection are compared with the data of [4] (1), in which the problem of convection near an impermeable surface was solved for two values of A.

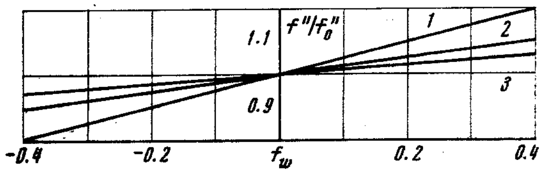

Fig. 2

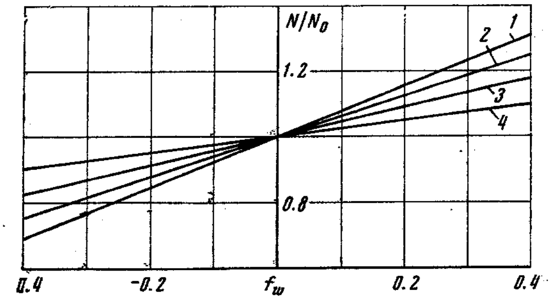

Fig. 3

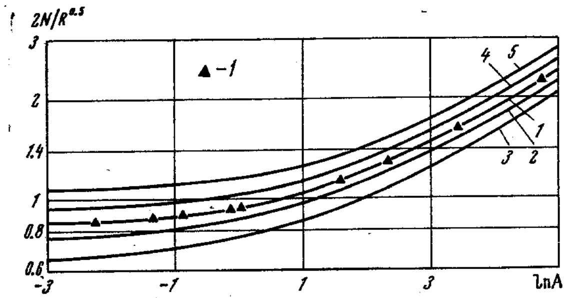

Fig. 4

The surface friction, under the condition that motion of the medium from the porous surface occurs, is given by the expression [1]

$$\tau_w = \mu\,(\partial u / \partial y)_{y=0} - \rho v_w U_\infty$$

This relationship in the transformed variables permits obtaining the dimensionless friction coefficient

$$c_f R^{0.5} = 0.5 f''(0) + (m+1) f_w \quad (c_f = 2\tau_w / (\rho U_\infty^2)). \tag{12}$$

To calculate the surface friction by Eq. (12) the value of $f''(0)$ was calculated with a computer for various parameter values, and is presented in Fig. 2 in the form of the ratio $f_0''/f''_0(0)$ at $P = 0.7$, where $f_0''(0)$ is taken for a nonpermeable surface, and for 1, $A = 0$; 2, $A = 1$; 3, $A = 10$.

The thermal flux with convection on the vertical surface is defined by the equation

$$q = -\lambda\,(\partial T / \partial y)_{y=0} \tag{13}$$

where λ is the coefficient of thermal diffusivity.

The data on heat transfer are represented by the local heat transfer coefficient and the local Nusselt number

$$\alpha = q / (T_w - T_\infty), \quad N = (\alpha x) / \lambda$$

On the basis of this, the heat transfer will be

$$N = -0.5 R^{0.5} \theta'(0) \tag{14}$$

where the values of $\theta'(0)$ are calculated for various values of the defining parameters A and f_W.

In Fig. 3 results of heat transfer calculations with convection are shown: curve 1 corresponds to $A = 0$; 2, $A = 1$; 3, $A = 10$; and 4, $A = 100$, for $P = 0.7$. The Nusselt number N_0 characterizes heat transfer near an impermeable surface.

A comparison of the results (curve 1) of heat transfer on an impermeable surface for $P = 0.7$ with the data of [4] (1) is presented in Fig. 4. Calculated data for heat transfer are shown with draft ($f_W = -0.1$ and -0.3, curves 2, 3) and suction ($f_W = 0.1$ and 0.3, curves 4, 5). It is evident from analysis of these curves that the intensity of heat transfer increases with growth in free convection, which is explained by increase in convective velocities.

Starting from the fact that the value of the friction coefficient with convection is more than 5% different from the data with purely constrained flow, a criterion can be obtained for defining the limit of convection in calculating surface friction

$$G / R^2 \geqslant 0.03 f_w + 0.06 \tag{15}$$

To determine the limit of convection with draft (suction) in calculating heat transfer we have

$$G / R^2 \geqslant 0.57 f_w + 0.3 \tag{16}$$

With values of G/R^2 lower than those calculated in Eqs. (15) and (16) in problems of calculating friction and heat transfer, the effect of free convection can be neglected.

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